Proposed method to realize the p-wave superfluid state using a s-wave superfluid Fermi gas with a synthetic spin-orbit interaction

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Abstract

We theoretically propose an idea to reach the p-wave superfluid phase in an ultracold Fermi gas. The key of our idea is that the pairing symmetry of a Fermi superfluid is fully dominated by the symmetry of the superfluid order parameter, which is essentially given by the product of a pair amplitude and a pairing interaction. Noting this, in our proposal, we first prepare a p-wave pair amplitude by, not using a p-wave interaction, but using the phenomenon that a p-wave pair amplitude is induced in an s-wave superfluid Fermi gas with antisymmetric spin-orbit interaction. In this case, although the system is still in the s-wave superfluid state with the s-wave superfluid order parameter, when one suddenly replaces the s-wave interaction by an appropriate p-wave one (which is possible in cold Fermi gases by using a Feshbach resonance technique), the product of the p-wave interaction and the p-wave pair amplitude that has already been prepared in the spin-orbit coupled s-wave superfluid state immediately gives a finite p-wave superfluid order parameter. Thus, at least just after this manipulation, the system is in the p-wave superfluid state, being characterized by the artificially produced p-wave superfluid parameter. In this paper, to assess our idea, we evaluate the p-wave pair amplitude in a spin-orbit coupled s-wave superfluid Fermi gas at $T = 0$. We determine the region where a large p-wave pair amplitude is obtained in the phase diagram with respect to the strengths of the s-wave pairing interaction and the spin-orbit coupling. We also discuss the accessibility of this optimal region on the viewpoint of the superfluid phase transition temperature. Since the achievement of a p-wave superfluid Fermi gas is one of the most crucial issues in cold atom physics, our proposal would be useful for this exciting challenge.

I. INTRODUCTION

Since the achievement of the s-wave superfluid phase transition in ^{40}K[1] and ^{6}Li[2–4] Fermi gases, the possibility of the p-wave superfluid state has extensively been discussed as the next challenge in cold Fermi gas physics[5–19]. Although no one has succeeded in this attempt, p-wave Feshbach resonances have already been discovered in ^{40}K[20–23] and ^{6}Li[24–26]. Thus, once a p-wave superfluid Fermi gas is obtained, as in the s-wave case[29–39], one can systematically study this unconventional Fermi superfluid from the weak-coupling regime to the strong-coupling limit, by adjusting the threshold energy of a p-wave Feshbach resonance[27, 28]. Since this pairing state has also been discussed in various fields, such as liquid ^{3}He[40–42], metallic superconductivity[43–45], as well as neutron star[46], the realization of a tunable p-wave superfluid Fermi gas would also give a big impact in these research fields.

In the current stage of research toward the realization of a p-wave superfluid Fermi gas, one serious difficulty is that, although a p-wave attractive interaction is necessary for this pairing state, it also causes the so-called three-body loss[9, 47, 48], as well as the dipolar relaxation[23], leading to very short lifetime of p-wave pairs[49]. Thus, although one can prepare p-wave molecules[23, 50–52], they are soon destroyed, before the p-wave condensate grows. Thus, at this stage, it is crucial to overcome this difficulty.

The purpose of this paper is to propose a possible route to reach the p-wave superfluid phase in an ultracold Fermi gas. To explain our idea in a simple manner, we first note that any Fermi superfluid is characterized by a superfluid order parameter $\Delta_{\sigma\sigma'}(\boldsymbol{p})$, being consisting of the product of the pairing interaction $U(\boldsymbol{p}, \boldsymbol{p}')$ and the pair amplitude $\Phi_{\sigma,\sigma'}(\boldsymbol{p}) = \langle c_{\boldsymbol{p},\sigma}c_{-\boldsymbol{p},\sigma'}\rangle$ as

$$\begin{aligned}
\Delta_{\sigma,\sigma'}(\boldsymbol{p}) &= \sum_{\boldsymbol{p}'} U(\boldsymbol{p}, \boldsymbol{p}')\langle c_{\boldsymbol{p}',\sigma}c_{-\boldsymbol{p}',\sigma'}\rangle \\
&= \bar{U}\gamma_{\boldsymbol{p}} \sum_{\boldsymbol{p}'} \gamma_{\boldsymbol{p}'}\Phi_{\sigma,\sigma'}(\boldsymbol{p}').
\end{aligned} \tag{1}$$

Here, $c_{\boldsymbol{p},\sigma}$ is the annihilation operator of a fermion with momentum $\boldsymbol{p}$ and (pseudo) spin $\sigma = \uparrow, \downarrow$. In the second line in Eq. (1), we have assumed the separable interaction $U(\boldsymbol{p}, \boldsymbol{p}') = \bar{U}\gamma_{\boldsymbol{p}}\gamma_{\boldsymbol{p}'}$ (where $\bar{U}$ is a coupling constant and $\gamma_{\boldsymbol{p}}$ is a basis function). When the pair amplitude $\Phi_{\sigma,\sigma'}(\boldsymbol{p})$ has the same symmetry as that of the basis function $\gamma_{\boldsymbol{p}}$, we obtain a finite value of the superfluid order parameter $\Delta_{\sigma,\sigma'}(\boldsymbol{p}) \propto \gamma_{\boldsymbol{p}}$.

In a Fermi superfluid, the pair amplitude $\Phi_{\sigma,\sigma'}(\boldsymbol{p})$ is usually produced by the pairing interaction $U(\boldsymbol{p}, \boldsymbol{p}')$ of the system. Thus, current experiments usually deal with a Fermi gas with a p-wave pairing interaction from the beginning. Although this approach seems the shortest way to reach the p-wave superfluid state, at present, no one has succeeded in this attempt, because of the above mentioned serious problem.

Instead of this ordinary approach, we consider an alternative route. That is, we first prepare only a p-wave pair amplitude in a system with *no p-wave interaction*. Of course, even when we prepare this quantity, the system is still not a p-wave Fermi superfluid, because the p-wave superfluid order parameter is absent due to the vanishing p-wave interaction. However, during this preparation, various difficulties originating from the p-wave interaction can be avoided. After this preparation, when the s-wave pairing interaction is suddenly replaced by a p-wave one $U_p(\boldsymbol{p}, \boldsymbol{p}') = \bar{U}_p \gamma_{\boldsymbol{p}} \gamma_{\boldsymbol{p}'}$ (where the p-wave basis function $\gamma_{\boldsymbol{p}}$ is chosen so that the momentum summation in Eq. (1) can be finite), the product of this interaction and the p-wave pair amplitude that has been prepared in advance immediately gives a finite value of the p-wave superfluid order parameter $\Delta_{\sigma,\sigma'}(\boldsymbol{p}) \propto \gamma_{\boldsymbol{p}}$. Thus, at least just after this manipulation, the system is regarded as a p-wave Fermi superfluid, by definition.

In a sense, the superfluid order parameter is artificially produced in our approach, so that the resulting p-wave superfluid Fermi gas would initially be in the non-equilibrium state. In addition, when the p-wave interaction is turned on, the above mentioned problem occurs, so that the p-wave superfluid phase might eventually be destroyed. However, the p-wave superfluid order parameter would continue to exist for a while, during the decay and relaxation of the system. This is quite different from the ordinary approach, where the system is destroyed before the reach of the p-wave superfluid phase.

To prepare a p-wave pair amplitude in our approach, the recent artificial gauge field would be useful[53–61]. This sophisticated technique enables us to introduce an antisymmetric spin-orbit interaction to an ultracold atom gas, leading to the broken spatial inversion symmetry. In this case, the parity is no longer a good quantity to classify the spatial pairing symmetry of the Fermi superfluid order parameter[62, 63]. Since the pair wavefunction must be antisymmetric with respect to the exchange of two fermions, the parity mixing naturally leads to the admixture of the spin singlet and spin triplet states[62, 63]. As a result, even in a s-wave Fermi superfluid, the pair amplitude may have the spin-triplet component. Thus, using this, one may prepare a p-wave pair amplitude without relying on a p-wave interaction.

Indeed, Refs.[64, 65] explicitly show that the p-wave pair amplitude is induced in a s-wave superfluid Fermi gas in the presence of the Rashba spin-orbit interaction[66], described by the Hamiltonian,

$$H_{\text{Rashba}} = \lambda_{\text{Rashba}} \sum_{\boldsymbol{p}} [p_x \hat{\sigma}_y^{\alpha,\alpha'} - p_y \hat{\sigma}_x^{\alpha,\alpha'}] c_{\boldsymbol{p},\alpha}^\dagger c_{\boldsymbol{p},\alpha'}, \tag{2}$$

where $\hat{\sigma}_i (i = x, y, z)$ are Pauli matrices acting on spin space and λ_{Rashba} is a spin-orbit coupling constant. At present, a simple antisymmetric spin-orbit interaction has been realized in ^{6}Li[58] and ^{40}K[59] Fermi gases and various ideas to synthesize more complicated spin-orbit couplings have also been proposed[60, 61].

Once a p-wave pair amplitude is induced in a s-wave superfluid Fermi gas, the p-wave superfluid order parameter can be produced by the sudden replacement of the s-wave interaction with a p-wave one, by tuning an external magnetic field from a s-wave Feshbach resonance field to a p-wave Feshbach resonance field. After this manipulation, the s-wave superfluid order parameter immediately vanishes, because of the vanishing s-wave interaction. Although the s-wave pair amplitude still remains, since the pairing symmetry of a Fermi superfluid is determined by the symmetry of the order parameter, this system is in the p-wave superfluid state.

We note that the admixture of the spin-singlet pairing and spin-triplet pairing by an antisymmetric spin-orbit interaction has also been discussed in non-centrosymmetric superconductivity[63]. For example, Ref.[67] points out the importance of this admixture to understand the anomalous behavior of the penetration depth observed in the non-centrosymmetric superconductor Li$_2$PtB$_3$.

Our idea is somehow related to the proximity effect in a superconductor-normal metal (S-N) junction[68]. In this case, the pair amplitude in the S side penetrates into the N side, so that, when it couples with an interaction existing in the N side, a finite superconducting order parameter appears in the N side. Since the pair amplitude in the N side is fully supplied from the S side, even when the interaction in the N side is *repulsive*, this proximity-induced superconducting state is obtained[69].

In this paper, to assess our idea, we investigate a s-wave superfluid Fermi gas with an antisymmetric spin-orbit interaction. Using the strong-coupling theory developed by Eagles[29] and Leggett[30, 31] (which is sometimes referred to the BCS-Leggett theory in the literature), we examine how large a p-wave pair amplitude is induced by a spin-orbit interaction

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at $T = 0$. We determine the region where a large p-wave amplitude is obtained, in the phase diagram with respect to the strengths of the s-wave interaction and the spin-orbit coupling. We also examine the accessibility of this "optimal region" within the current experimental technique. For this purpose, employing the strong-coupling theory developed by Nozières and Schmitt-Rink (NSR)[32], we calculate the superfluid phase transition temperature T_c around this region. We briefly note that T_c in a spin-orbit coupled ultracold Fermi gas has recently been discussed in the BCS (Bardeen-Cooper-Schrieffer)-BEC (Bose-Einstein condensation) region within the framework of a T-matrix approximation[70, 71], where pairing fluctuations are treated within the static approximation[36]. In this paper, we do not employ the static approximation, but fully take into account dynamical properties of pairing fluctuations within the NSR scheme.

For the time evolution of the system after the introduction of a p-wave interaction, we need to deal with a non-equilibrium Fermi gas, which we will separately discuss in our future paper.

This paper is organized as follows. In Sec. II, we explain the outline of our formulations at $T = 0$, as well as at T_c, in the BCS-BEC crossover region. Here, we also introduce the condensate fraction, as a useful quantity to evaluate the magnitude of the pair amplitude. In Sec. III, we discuss how large a p-wave pair amplitude is induced at $T = 0$, when the inversion symmetry of the system is broken by a spin-orbit interaction. We clarify the condition to obtain a large p-wave pair amplitude. We also calculate the superfluid phase transition temperature T_c, to examine the accessibility of this condition within the current experimental technique. Throughout this paper, we set $\hbar = k_B = 1$, and the system volume is taken to be unity, for simplicity.

II. FORMULATION

We consider a spin-orbit coupled uniform two-component Fermi gas with an s-wave pairing interaction. To explain our formulation at $T = 0$ within the framework of the BCS-Leggett theory[29–31], as well as the formulation at T_c within the framework of the NSR theory[32], in a unified manner, the functional integral formalism is convenient[33, 72]. The

partition function in this formalism is given by

$$\mathcal{Z} = \int \prod_{\sigma} \mathcal{D}\bar{\Psi}_{\sigma}\mathcal{D}\Psi_{\sigma}e^{-S}, \tag{3}$$

where the action S has the form,

$$S = \int dx \left[\sum_{\sigma} \bar{\Psi}_{\sigma}(x) \left[\frac{\partial}{\partial \tau} + \frac{\hat{\boldsymbol{p}}^2}{2m} - \mu \right] \Psi_{\sigma}(x) + \sum_{\sigma,\sigma'} \bar{\Psi}_{\sigma}(x) h_{\mathrm{so}}^{\sigma,\sigma'} \Psi_{\sigma'}(x) - U_s \bar{\Psi}_{\uparrow}(x)\bar{\Psi}_{\downarrow}(x)\Psi_{\downarrow}(x)\Psi_{\uparrow}(x) \right]. \tag{4}$$

Here, $x = (\boldsymbol{r}, \tau)$ and $\int dx = \int_0^{\beta} d\tau \int d\boldsymbol{r}$, where $\beta = 1/T$. $\Psi_{\sigma}(\boldsymbol{r}, \tau)$ and $\bar{\Psi}_{\sigma}(\boldsymbol{r}, \tau)$ are a Grassmann variable and its conjugate, describing Fermi atoms with the atomic mass m and pseudospin $\sigma = \uparrow, \downarrow$, specifying two atomic hyperfine states. $\hat{\boldsymbol{p}} = -i\nabla$ is the momentum operator in real space and μ is the Fermi chemical potential. The s-wave pairing interaction $-U_s$ (< 0) is assumed to be tunable by an s-wave Feshbach resonance. As usual, we measure the interaction strength in terms of the s-wave scattering length a_s, given by

$$\frac{4\pi a_s}{m} = -\frac{U_s}{1 - U_s \sum_{\boldsymbol{p}}^{p_c} \frac{1}{2\varepsilon_{\boldsymbol{p}}}}, \tag{5}$$

where $\varepsilon_{\boldsymbol{p}} = p^2/(2m)$ and p_c is a cutoff momentum.

The antisymmetric spin-orbit interaction $\hat{h}_{\mathrm{so}} = \{h_{\mathrm{so}}^{\sigma,\sigma'}\}$ with linear-momentum dependence in Eq. (4) generally has the form[73–76]

$$\hat{h}_{\mathrm{so}} = \sum_{i,j} \hat{p}_i \lambda_{i,j} \hat{\sigma}_j, \tag{6}$$

where the 3×3-matrix $\hat{\lambda} = \{\lambda_{i,j}\}$ ($i, j = x, y, z$) describes spin-orbit coupling strengths. However, this paper deals with the simpler version, $\hat{\lambda} = \mathrm{diag}[\lambda_{\perp}, \lambda_{\perp}, \lambda_z]$, that is,

$$\hat{h}_{\mathrm{so}} = \lambda_{\perp}[\hat{p}_x \hat{\sigma}_x + \hat{p}_y \hat{\sigma}_y] + \lambda_z \hat{p}_z \hat{\sigma}_z. \tag{7}$$

Here, we take $\lambda_{\perp}, \lambda_z \geq 0$ without loss of generality. Although Eq. (7) cannot describe all possible spin-orbit interactions, it still covers some typical cases that have recently been discussed in cold atom physics. The Rashba-type spin-orbit interaction in Eq. (2) is obtained by setting $\lambda_z = 0$ and rotating the momentum space by $\pi/2$ around the p_z axis. The single-component spin-orbit interaction $h_{\mathrm{so}} \sim \hat{p}_x \hat{\sigma}_y$, which has recently been synthesized in ^{6}Li[58] and ^{40}K[59] Fermi gases, is also obtained by setting $\lambda_z = 0$ and rotating the momentum space by $\pi/2$ around the p_y axis, which is followed by the rotation in the $(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ space by $\pi/2$ around the $\hat{\sigma}_x$ axis.

As usual, we introduce the Cooper pair field $\Delta(x)$ and its conjugate $\Delta^*(x)$, using the Hubbard-Stratonovich transformation[33]. Carrying out the functional integrals with respect to $\Psi_\sigma(x)$ and $\bar{\Psi}_\sigma(x)$, one has[33]

$$\mathcal{Z} = \int \mathcal{D}\Delta^* \mathcal{D}\Delta e^{-S_{\text{eff}}(\Delta,\Delta^*)}, \tag{8}$$

where the action $S_{\text{eff}}(\Delta, \Delta^*)$ is given by

$$S_{\text{eff}}(\Delta, \Delta^*) = \int dx \frac{|\Delta(x)|^2}{U_s} - \frac{1}{2} \text{Tr} \ln[-\hat{G}^{-1}]. \tag{9}$$

Here,

$$\hat{G}^{-1}(x, x') = \begin{pmatrix} -\dfrac{\partial}{\partial\tau} - \left[\dfrac{\hat{\boldsymbol{p}}^2}{2m} - \mu\right] - \hat{h}_{\text{so}} & i\hat{\sigma}_y\Delta(x) \\ -i\hat{\sigma}_y\Delta^*(x) & -\dfrac{\partial}{\partial\tau} + \left[\dfrac{\hat{\boldsymbol{p}}^2}{2m} - \mu\right] - \hat{h}_{\text{so}}^* \end{pmatrix} \delta(x - x') \tag{10}$$

is the inverse of the 4×4-matrix single-particle thermal Green's function $\hat{G}(x, x') = -\langle T_\tau\{\hat{\Psi}(x)\hat{\Psi}^\dagger(x')\}\rangle$ in the operator formalism, where

$$\hat{\Psi}(x) = \begin{pmatrix} \Psi_\uparrow(x) \\ \Psi_\downarrow(x) \\ \Psi_\uparrow^\dagger(x) \\ \Psi_\downarrow^\dagger(x) \end{pmatrix} \tag{11}$$

is the four component Nambu field[77, 78].

A. p-wave pair amplitude in a spin-orbit coupled s-wave superfluid state at $T = 0$

We evaluate the p-wave pair amplitude at $T = 0$ within the framework of the BCS-Leggett[29–31] theory. In the functional integral formalism, this strong-coupling theory is simply reproduced by approximately evaluating the functional integral in the partition function $\mathcal{Z}$ in Eq. (8) by the value at the saddle point solution (Δ_s), which is determined from the equation,

$$0 = \left(\frac{\delta S_{\text{eff}}}{\delta\Delta^*(x)}\right)_{\Delta(x)=\Delta^*(x)=\Delta_s} = \frac{\Delta_s}{U_s} + \frac{1}{4\beta}\sum_{\boldsymbol{p},\omega_n} \text{Tr}\left[[\hat{\rho}_y + i\hat{\rho}_x]\hat{\sigma}_y\hat{G}^{\text{MF}}(\boldsymbol{p}, i\omega_n)\right], \tag{12}$$

where ω_n is the fermion Matsubara frequency and $\hat{\rho}_j$ $(j = x, y, z)$ are Pauli matrices acting on particle-hole space. (Note that $\hat{\sigma}_j$ $(j = x, y, z)$ act on spin space.) In Eq. (12), $\hat{G}^{\text{MF}}(\boldsymbol{p}, i\omega_n)$

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is the Fourier transformed Green's function in the mean-field BCS approximation, given by

$$
\begin{aligned}
\hat{G}^{\mathrm{MF}}(\boldsymbol{p}, i\omega_n) &= \frac{1}{i\omega_n - [\xi_{\boldsymbol{p}} + \boldsymbol{p}_\lambda \cdot \hat{\boldsymbol{\tau}}]\hat{\rho}_z - \hat{\rho}_y \hat{\sigma}_y \Delta_s} \\
&= -\frac{1}{2} \sum_{\alpha=\pm} \frac{i\omega_n + [\xi_{\boldsymbol{p}} + \boldsymbol{p}_\lambda \cdot \hat{\boldsymbol{\tau}}]\rho_z + \Delta_s \hat{\rho}_y \hat{\sigma}_y}{\omega_n^2 + (E_{\boldsymbol{p}}^\alpha)^2} \Big[\mathbf{1} + \alpha \frac{\mathbf{p}_\lambda \cdot \hat{\boldsymbol{\tau}}}{|\mathbf{p}_\lambda|} \Big],
\end{aligned}
\tag{13}
$$

where $\xi_{\boldsymbol{p}} = \varepsilon_{\boldsymbol{p}} - \mu = p^2/(2m) - \mu$ is the kinetic energy of a Fermi atom, measured from the Fermi chemical potential μ. $E_{\boldsymbol{p}}^\alpha = \sqrt{(\xi_{\boldsymbol{p}}^\alpha)^2 + \Delta_s^2}$ describes the Bogoliubov single particle excitations, where $\xi_{\boldsymbol{p}}^\alpha = \xi_{\boldsymbol{p}} + \alpha|\boldsymbol{p}_\lambda|$ with $\boldsymbol{p}_\lambda = (\lambda_\perp p_x, \lambda_\perp p_y, \lambda_z p_z)$. In Eq. (13), $\hat{\boldsymbol{\tau}} = (\hat{\rho}_z \hat{\sigma}_x, \hat{\sigma}_y, \hat{\rho}_z \hat{\sigma}_z)$ is the spin operator under the four component Nambu representation[77, 78]. Summing up the Matsubara frequencies in Eq. (12), we obtain the BCS gap equation in the presence of spin-orbit interaction. At $T = 0$, it has the form,

$$
1 = \frac{U_s}{2} \sum_{\boldsymbol{p}, \alpha=\pm} \frac{1}{2E_{\boldsymbol{p}}^\alpha}.
\tag{14}
$$

Eliminating the ultraviolet divergence from Eq. (14), we obtain[30, 33]

$$
1 = -\frac{4\pi a_s}{m} \sum_{\boldsymbol{p}} \Big[\frac{1}{2} \sum_{\alpha=\pm} \frac{1}{2E_{\boldsymbol{p}}^\alpha} - \frac{1}{2\varepsilon_{\boldsymbol{p}}} \Big].
\tag{15}
$$

We solve the renormalized gap equation (15), together with the equation for the number N of Fermi atoms, which is obtained from the mean-field thermodynamic potential $\Omega_{\mathrm{MF}} = TS_{\mathrm{eff}}(\Delta_s, \Delta_s)$ as

$$
N = -\frac{\partial \Omega_{\mathrm{MF}}}{\partial \mu} = \frac{1}{2\beta} \sum_{\boldsymbol{p}, \omega_n} \mathrm{Tr}\Big[\rho_z \hat{G}^{\mathrm{MF}}(\boldsymbol{p}, i\omega_n) e^{i\rho_z \omega_n \delta} \Big] = \sum_{\boldsymbol{p}} \Big[1 - \frac{1}{2} \sum_{\alpha=\pm} \frac{\xi_{\boldsymbol{p}}^\alpha}{E_{\boldsymbol{p}}^\alpha} \Big]
\tag{16}
$$

(where δ is an infinitesimally small positive number), to self-consistently determine Δ_s and μ.

The pair amplitude $\Phi(\boldsymbol{p}, S, S_z)$ with total spin S and its z-component S_z is obtained from the off-diagonal components of the Green's function $\hat{G}^{\mathrm{MF}}(\boldsymbol{p}, i\omega_n)$ in Eq. (13). Noting that $\hat{G}^{\mathrm{MF}}(\boldsymbol{p}, \tau)$ in the operator formalism has the form

$$
\hat{G}^{\mathrm{MF}}(\boldsymbol{p}, \tau) = -\Big\langle \mathrm{T}_\tau\Big\{ \begin{pmatrix} c_{\boldsymbol{p},\uparrow}(\tau) \\ c_{\boldsymbol{p},\downarrow}(\tau) \\ c_{-\boldsymbol{p},\uparrow}^\dagger(\tau) \\ c_{-\boldsymbol{p},\downarrow}^\dagger(\tau) \end{pmatrix} (c_{\boldsymbol{p},\uparrow}^\dagger(0), c_{\boldsymbol{p},\downarrow}^\dagger(0), c_{-\boldsymbol{p},\uparrow}(0), c_{-\boldsymbol{p},\downarrow}(0)) \Big\}\Big\rangle,
\tag{17}
$$

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one finds,

$$
\begin{aligned}
\Phi(\boldsymbol{p},0,0) &= \frac{1}{2}\Big[\langle c_{\boldsymbol{p},\uparrow}c_{-\boldsymbol{p},\downarrow}\rangle - \langle c_{\boldsymbol{p},\downarrow}c_{-\boldsymbol{p},\uparrow}\rangle\Big] = -\frac{1}{2}\Big[\hat{G}_{14}^{\mathrm{MF}}(\boldsymbol{p},\tau=0) - \hat{G}_{23}^{\mathrm{MF}}(\boldsymbol{p},\tau=0)\Big] \\
&= -\frac{1}{2}\sum_{\alpha=\pm}\frac{\Delta_s}{2E_{\boldsymbol{p}}^{\alpha}}, \\
\Phi(\boldsymbol{p},1,1) &= \langle c_{\boldsymbol{p},\uparrow}c_{-\boldsymbol{p},\uparrow}\rangle = -\hat{G}_{13}^{\mathrm{MF}}(\boldsymbol{p},\tau=0) = \frac{\lambda_\perp}{2}\frac{p_x-ip_y}{|\boldsymbol{p}_\lambda|}\sum_{\alpha=\pm}\frac{\alpha\Delta_s}{2E_{\boldsymbol{p}}^{\alpha}}, \\
\Phi(\boldsymbol{p},1,0) &= \frac{1}{2}\Big[\langle c_{\boldsymbol{p},\uparrow}c_{-\boldsymbol{p},\downarrow}\rangle + \langle c_{\boldsymbol{p},\downarrow}c_{-\boldsymbol{p},\uparrow}\rangle\Big] = -\frac{1}{2}\Big[\hat{G}_{14}^{\mathrm{MF}}(\boldsymbol{p},\tau=0) + \hat{G}_{23}^{\mathrm{MF}}(\boldsymbol{p},\tau=0)\Big] \\
&= -\frac{\lambda_z}{2}\frac{p_z}{|\boldsymbol{p}_\lambda|}\sum_{\alpha=\pm}\frac{\alpha\Delta_s}{2E_{\boldsymbol{p}}^{\alpha}}, \\
\Phi(\boldsymbol{p},1,-1) &= \langle c_{\boldsymbol{p},\downarrow}c_{-\boldsymbol{p},\downarrow}\rangle = -\hat{G}_{24}^{\mathrm{MF}}(\boldsymbol{p},\tau=0) = -\frac{\lambda_\perp}{2}\frac{p_x+ip_y}{|\boldsymbol{p}_\lambda|}\sum_{\alpha=\pm}\frac{\alpha\Delta_s}{2E_{\boldsymbol{p}}^{\alpha}}. \quad (18)
\end{aligned}
$$

Equation (7) clearly shows that the antisymmetric spin-orbit interaction induces the p-wave pair amplitudes $\Phi(\boldsymbol{p}, S=1, S_z=\pm1,0)$. However, we emphasize that the p-wave superfluid order parameter is still absent due to the vanishing p-wave pairing interaction. The system is thus in the s-wave superfluid state which is characterized by the s-wave order parameter,

$$
\Delta_s = -U_s \sum_{\boldsymbol{p}} \Phi(\boldsymbol{p},0,0). \quad (19)
$$

At $t=0$, we suddenly replace the s-wave pairing interaction by the p-wave one[5, 6, 19],

$$
H_{p-\mathrm{wave}} = -U_p \sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} \boldsymbol{p}\cdot\boldsymbol{p}' c_{\boldsymbol{p}+\boldsymbol{q}/2,\sigma}^{\dagger} c_{-\boldsymbol{p}+\boldsymbol{q}/2,\sigma'}^{\dagger} c_{-\boldsymbol{p}'+\boldsymbol{q}/2,\sigma'} c_{\boldsymbol{p}'+\boldsymbol{q}/2,\sigma}. \quad (20)
$$

Then, while the s-wave order parameter immediately vanishes due to the vanishing s-wave interaction, a p-wave order parameter $\Delta_p(\boldsymbol{p}, S_z, t>0)$ become finite, so that, by definition, the system is in the p-wave superfluid state. In particular, the p-wave order parameter just after this manipulation ($t=+0$) is simply given by the momentum summation of the product of the p-wave pairing interaction U_p and a p-wave pair amplitude $\Phi(\boldsymbol{p}, S=1, S_z)|_{t=-\delta}$ that has already been prepared in the s-wave superfluid state. For example, when $\lambda_\perp=0$ and $\lambda_z \neq 0$, one has

$$
\Delta_p(\boldsymbol{p}, S_z=0, t=+0) = -U_p p_z \sum_{\boldsymbol{p}'} p_z' \Phi(\boldsymbol{p}',1,0)|_{t=-\delta} \propto p_z. \quad (21)
$$

We note that, although all the pair amplitudes $\Phi(\boldsymbol{p}, S, S_z)$ in Eq. (18) are proportional to the s-wave order parameter Δ_s, it does not mean that they immediately disappear, when

Δ_s vanishes. As well known in the proximity effect[68, 69], even when the pair amplitude penetrates into the normal metal with no interaction, it continues to exist until they are destroyed by external perturbations, such as thermal fluctuations and impurity scatterings. Thus, in the present case, the pair amplitudes in Eq. (18) should be regarded as the "initial values", in considering the time evolutions of $\Phi(\boldsymbol{p}, S, S_z, t \geq 0)$ after the sudden change of the pairing interaction from the s-wave one to the p-wave one.

To evaluate the strength of the p-wave pair amplitude, the condensate fraction[30, 79–82] is a useful quantity. It is deeply related to the pair amplitude and physically describes the number of Bose-condensed Cooper pairs. In addition, it has widely been used in detecting the superfluid phase transition in ultracold Fermi gases[1–4]. In the present spin-orbit coupled case, the total condensate fraction $N_{\mathrm{c}}^{\mathrm{t}}$ is given by[31]

$$N_{\mathrm{c}}^{\mathrm{t}} = \frac{1}{2} \sum_{\boldsymbol{p},\sigma,\sigma'} |\langle c_{\boldsymbol{p},\sigma} c_{-\boldsymbol{p},\sigma'} \rangle|^2 = N_{\mathrm{c}}(S = 0, S_z = 0) + \sum_{S_z=-1}^{1} N_{\mathrm{c}}(S = 1, S_z). \tag{22}$$

Here,

$$N_{\mathrm{c}}(S = 0, S_z = 0) = \sum_{\boldsymbol{p}} |\Phi(\boldsymbol{p}, 0, 0)|^2 = \frac{\Delta_s^2}{16} \sum_{\boldsymbol{p}} \left(\sum_{\alpha=\pm} \frac{1}{E_{\boldsymbol{p}}^{\alpha}} \right)^2 \tag{23}$$

is the s-wave condensate fraction. Equation (22) also involves the p-wave components $N_{\mathrm{c}}(S = 1, S_z = \pm 1, 0)$, given by

$$
\begin{aligned}
N_{\mathrm{c}}(1, 1) &= \frac{1}{2} \sum_{\boldsymbol{p}} |\Phi(\boldsymbol{p}, 1, 1)|^2 = \frac{\Delta_s^2}{32} \sum_{\boldsymbol{p}} \frac{\lambda_\perp^2 (p_x^2 + p_y^2)}{|\boldsymbol{p}_\lambda|^2} \left(\sum_{\alpha=\pm} \frac{\alpha}{E_{\boldsymbol{p}}^{\alpha}} \right)^2, \\
N_{\mathrm{c}}(1, 0) &= \sum_{\boldsymbol{p}} |\Phi(\boldsymbol{p}, 1, 0)|^2 = \frac{\Delta_s^2}{16} \sum_{\boldsymbol{p}} \frac{\lambda_z^2 p_z^2}{|\boldsymbol{p}_\lambda|^2} \left(\sum_{\alpha=\pm} \frac{\alpha}{E_{\boldsymbol{p}}^{\alpha}} \right)^2, \\
N_{\mathrm{c}}(1, -1) &= \frac{1}{2} \sum_{\boldsymbol{p}} |\Phi(\boldsymbol{p}, 1, -1)|^2 = \frac{\Delta_s^2}{32} \sum_{\boldsymbol{p}} \frac{\lambda_\perp^2 (p_x^2 + p_y^2)}{|\boldsymbol{p}_\lambda|^2} \left(\sum_{\alpha=\pm} \frac{\alpha}{E_{\boldsymbol{p}}^{\alpha}} \right)^2.
\end{aligned}
\tag{24}
$$

Substituting Eqs. (23) and (24) into Eq. (22), one has

$$N_{\mathrm{c}}^{\mathrm{t}} = \frac{\Delta_s^2}{8} \sum_{\boldsymbol{p},\alpha=\pm} \left(\frac{1}{E_{\boldsymbol{p}}^{\alpha}} \right)^2. \tag{25}$$

In Sec. III, we will numerically calculate $N_{\mathrm{c}}(S = 1, S_z = \pm 1, 0)$ to examine how large the p-wave components are induced by a spin-orbit interaction.

B. Superfluid phase transition temperature and effects of spin-orbit interaction

To evaluate T_c and effects of a spin-orbit interaction in the BCS-BEC crossover region, we include pairing fluctuations within the framework of the strong-coupling theory developed by Nozières and Schmitt-Rink[32]. In the functional integral formalism, the T_c equation in the NSR scheme is immediately obtained from the saddle point condition, $\delta S_{\text{eff}}/\delta \Delta^*(x)|_{\Delta(x)=\Delta^*(x)=0} = 0$[33]. After the renormalization, it is given by,

$$1 = -\frac{4\pi a_s}{m} \sum_{\boldsymbol{p}} \Big[\frac{1}{2} \sum_{\alpha=\pm} \frac{1}{2\xi_{\boldsymbol{p}}^\alpha} \tanh \frac{\xi_{\boldsymbol{p}}^\alpha}{2T} - \frac{1}{2\varepsilon_{\boldsymbol{p}}} \Big]. \tag{26}$$

As usual, we solve the T_c equation (26), together with the equation for the number N of Fermi atoms, to self-consistently determine T_c and the Fermi chemical potential μ. The NSR number equation includes pairing fluctuations within the Gaussian fluctuation level, which is derived from the identity $N = -\partial\Omega_{\text{NSR}}/\partial\mu$. The NSR thermodynamic potential Ω_{NSR} is obtained by expanding the action S_{eff} in Eq. (9) around $\Delta(x) = 0$ to the quadratic order, which is followed by carrying out functional integrals with respect to $\Delta(x)$ and $\Delta^*(x)$. The result is

$$N = N_{\text{free}} - T\frac{\partial}{\partial\mu} \sum_{\boldsymbol{q},\nu_n} \ln \Big[1 + \frac{4\pi a_s}{m} \Big[\Pi(\boldsymbol{q}, i\nu_n) - \sum_{\boldsymbol{p}} \frac{1}{2\varepsilon_{\boldsymbol{p}}} \Big] \Big] e^{i\nu_n\delta}, \tag{27}$$

where we have eliminated the ultraviolet divergence by employing the renormalization prescription[33] and δ is infinitely small positive real number. In Eq. (27), $N_{\text{free}} = \sum_{\boldsymbol{p},\alpha=\pm} f(\xi_{\boldsymbol{p}}^\alpha)$ is the number of Fermi atoms in the absence of the pairing interaction U_s, where $f(x)$ is the Fermi distribution function. The second term in Eq. (27) describes effects of pairing fluctuations, where

$$\Pi(\boldsymbol{q}, i\nu_n) = \frac{1}{4} \sum_{\boldsymbol{p},\alpha,\alpha'=\pm} \frac{1 - f(\xi_{\boldsymbol{p}+\boldsymbol{q}/2}^\alpha) - f(\xi_{\boldsymbol{p}-\boldsymbol{q}/2}^{\alpha'})}{\xi_{\boldsymbol{p}+\boldsymbol{q}/2}^\alpha + \xi_{\boldsymbol{p}-\boldsymbol{q}/2}^{\alpha'} - i\nu_n} \Big[1 + \alpha\alpha' \frac{(\boldsymbol{p}_\lambda + \boldsymbol{q}_\lambda/2) \cdot (\boldsymbol{p}_\lambda - \boldsymbol{q}_\lambda/2)}{|\boldsymbol{p}_\lambda + \boldsymbol{q}_\lambda/2||\boldsymbol{p}_\lambda - \boldsymbol{q}_\lambda/2|} \Big] \tag{28}$$

is the lowest-order pair-correlation function in terms of the pairing interaction (where ν_n is the boson Matsubara frequency).
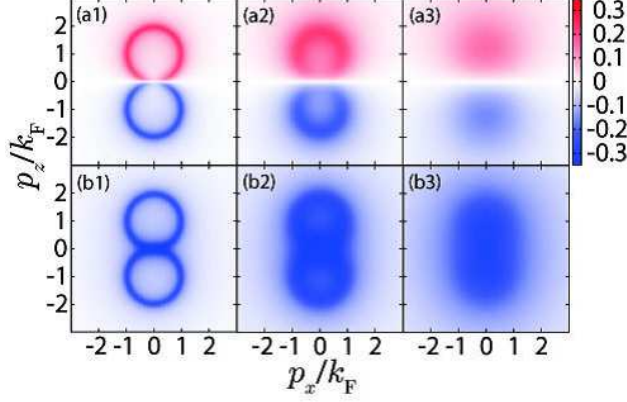
FIG. 1: (Color online) Calculated intensity of pair amplitude $\Phi(\boldsymbol{p}, S, S_z)$ at $T = 0$. We set $\lambda_\perp = 0$ and $\lambda_z/v_F = 1$ (where $v_F = k_F/m$ is the Fermi velocity with k_F being the Fermi momentum of a free Fermi gas in the absence of spin-orbit interaction). The upper and lower panels show the p-wave component $\Phi(\boldsymbol{p}, S = 1, S_z = 0)$ and the s-wave component $\Phi(\boldsymbol{p}, S = 0, S_z = 0)$, respectively. (a1) and (b1) $(k_F a_s)^{-1} = -1$. (a2) and (b2) $(k_F a_s)^{-1} = 0$. (a3) and (b3) $(k_F a_s)^{-1} = 1$. In these panels, we take $p_y = 0$.

III. p-WAVE PAIR AMPLITUDE INDUCED BY ANTISYMMETRIC SPIN-ORBIT INTERACTION

A.   single-component spin-orbit interaction ($\lambda_\perp = 0$, $\lambda_z \neq 0$)

Figure 1 shows the pair amplitude $\Phi(\boldsymbol{p}, S, S_z)$ in a spin-orbit coupled s-wave superfluid Fermi gas at $T = 0$ in the case of $\lambda_\perp = 0$ and $\lambda_z \neq 0$. Besides the s-wave pair amplitude shown in Figs. 1(b1)-(b3), Figs. 1(a1)-(a3) show that this single-component spin-orbit interaction induces the p-wave component with $(S, S_z) = (1, 0)$. While the s-wave component does not change its sign (See panels (b1)-(b3).), panels (a1)-(a3) clearly show the p_z-wave symmetry, as expected from the expression for $\Phi(\boldsymbol{p}, S = 1, S_z = 0)$ in Eq. (18).

In the weak-coupling BCS regime, one sees in Figs. 1(a1) and (b1) that both the p-wave and s-wave pair amplitudes are large around two circles. This is simply because, in the absence of the s-wave pairing interaction U_s, the present spin-orbit interaction gives two single-particle dispersions

$$\xi_{\boldsymbol{p}}^{\pm} = \frac{p_\perp^2}{2m} + \frac{(p_z \pm m\lambda_z)^2}{2m} - \tilde{\mu}_z, \tag{29}$$

where $p_\perp^2 = p_x^2 + p_y^2$ and $\tilde{\mu}_z = \mu + m\lambda_z^2/2$ is an effective Fermi chemical potential. These bands give two Fermi surfaces that are centered at $\boldsymbol{p} = (0, 0, \pm m\lambda_z)$ with the radius being equal to the Fermi momentum k_F in the absence of the spin-orbit interaction. In the weak-coupling BCS regime, since atoms near these two Fermi surfaces dominantly contribute to the Cooper-pair formation, the pair amplitude becomes large around them, as seen in Figs. 1(a1) and (b1). Since atoms away from the Fermi surfaces also contribute to the pair formation when the pairing interaction is strong, the circular structure is obscure in Figs. 1(a2) and (b2). In the strong-coupling BEC regime, the effective Fermi chemical potential $\tilde{\mu}_z$ is negative (See Fig. 2(c).), so that the Fermi surface no longer exists. As a result, the circular structure disappears in Figs. 1(a3) and (b3).

To evaluate the p-wave pair amplitude in a quantitative manner, we conveniently consider the condensate fraction $N_c(S, S_z)$ at $T = 0$. In the BCS side ($(k_F a_s)^{-1} \lesssim 0$), Fig. 2(a) shows that the p-wave component $N_c(S = 1, S_z = 0)$ increases with increasing the interaction strength, reflecting the increase of the magnitude of the s-wave superfluid order parameter Δ_s shown in Fig. 2(b). However, $N_c(S = 1, S_z = 0)$ decreases in the BEC side ($(k_F a_s)^{-1} \gtrsim 0$), although the s-wave condensate fraction $N_c(S = 0, S_z = 0)$ continues to increase. In this regime, the coupled equations (15) and (16) give,

$$\Delta_s = \sqrt{\frac{16}{3\pi(k_F a_s)}}\varepsilon_F,$$
$$\mu = -\frac{1}{2ma_s^2} - \frac{1}{2}m\lambda_z^2, \tag{30}$$

so that one obtains

$$N_c(1, 0) = \frac{N}{12}\left(\frac{\lambda_z}{v_F}\right)^2 (k_F a_s)^2,$$
$$N_c(0, 0) = \frac{N}{2} - N_c(1, 0). \tag{31}$$

This means that the strong-coupling BEC limit ($(k_F a_s)^{-1} \to \infty$) may be simply viewed as a gas of $N/2$ s-wave molecules, even in the presence of spin-orbit interaction.

When the spin-orbit interaction becomes strong, the magnitude of p-wave component $N_c(S = 1, S_z = 0)$ increases, as shown in Fig. 3(a). However, in the present case of single-component spin-orbit interaction, we should note that the total condensate fraction N_c^t, as well as the s-wave superfluid order parameter Δ_s, do not depend on the spin-orbit coupling strength λ_z. Indeed, when $\lambda_\perp = 0$ and $\lambda_z \neq 0$, the coupled equations (15) and (16) are
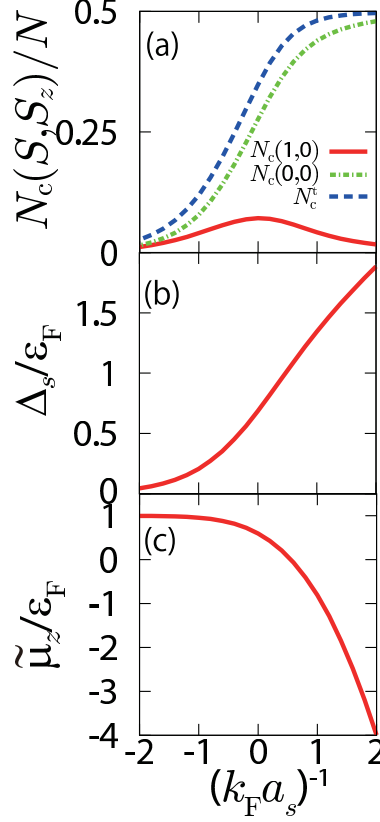
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FIG. 2: (Color online) (a) Calculated condensate fraction $N_c(S, S_z)$ in a spin-orbit coupled s-wave superfluid Fermi gas at $T = 0$. We take $\lambda_z/v_F = 1$ and $\lambda_\perp = 0$. In this single component case, one finds that $N_c(1, \pm 1) = 0$. N_c^t is the total condensate fraction in Eq. (22). (b) s-wave superfluid order parameter Δ_s, normalized by the Fermi energy $\varepsilon_F = k_F^2/(2m)$. (c) Effective Fermi chemical potential $\tilde{\mu}_z = \mu + m\lambda_z^2/2$.

reduced to the ordinary BCS-Leggett coupled equations for a s-wave Fermi superfluid *with no spin-orbit interaction*, as

$$1 = -\frac{4\pi a_s}{m} \sum_{\boldsymbol{p}} \left[\frac{1}{2\sqrt{\tilde{\xi}_{\boldsymbol{p}}^2 + \Delta_s^2}} - \frac{1}{2\varepsilon_{\boldsymbol{p}}} \right],$$

$$N = \sum_{\boldsymbol{p}} \left[1 - \frac{\tilde{\xi}_{\boldsymbol{p}}}{\sqrt{\tilde{\xi}_{\boldsymbol{p}}^2 + \Delta_s^2}} \right], \tag{32}$$

where $\tilde{\xi}_{\boldsymbol{p}} = \varepsilon_{\boldsymbol{p}} - \tilde{\mu}_z$. Thus, the self-consistent solutions $(\Delta_s, \tilde{\mu}_z)$ shown in Figs. 2(b) and (c) are independent of λ_z. As a result, the total condensate fraction N_c^t in Eq. (25) is also
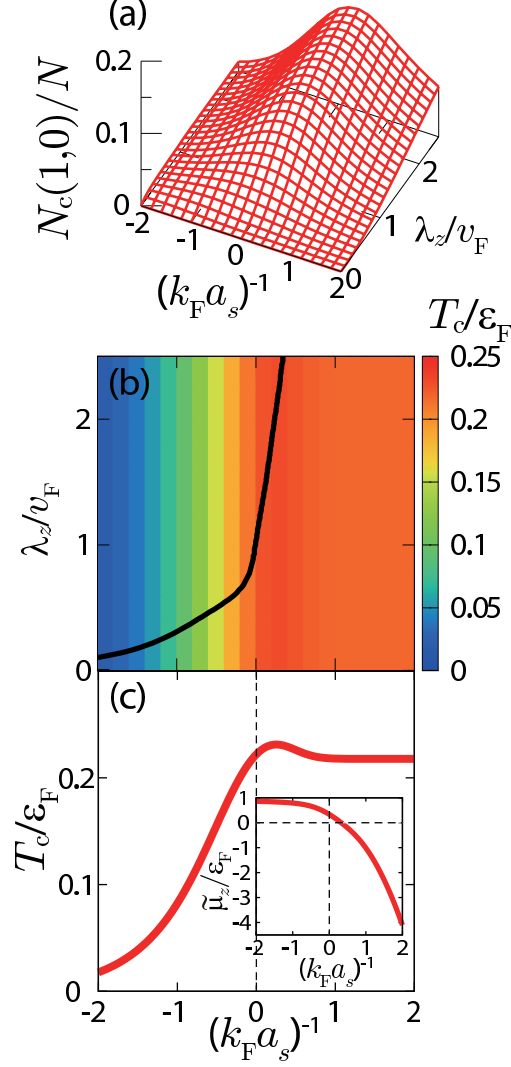
FIG. 3: (Color online) (a) Calculated p-wave condensate fraction $N_c(S = 1, S_z = 0)$ at $T = 0$ in the case of single-component spin-orbit interaction ($\lambda_\perp = 0$). (b) Peak position of $N_c(S = 1, S_z = 0)$ evaluated from the result shown in panel (a). The background intensity shows T_c. (c) Calculated T_c. The inset shows the effective Fermi chemical potential $\tilde{\mu}_z = \mu + m\lambda_z^2/2$.

λ_z-independent as

$$\frac{N_c^t}{N} = \frac{\Delta_s^2}{4N} \sum_{\boldsymbol{p}} \frac{1}{\tilde{\xi}_{\boldsymbol{p}}^2 + \Delta_s^2} = \frac{3\pi\Delta_s}{16\sqrt{2}\varepsilon_F^{3/2}} \sqrt{\tilde{\mu}_z + \sqrt{\tilde{\mu}_z^2 + \Delta_s^2}}, \tag{33}$$

where ε_F is the Fermi energy for a free Fermi gas without spin-orbit interaction. As a result, for a given interaction strength $(k_F a_s)^{-1}$, the s-wave condensate fraction $N_c(S = 0, S_z = 0) = N_c^t - N_c(S = 1, S_z = 0)$ becomes small when the spin-orbit interaction becomes strong, in spite of the fact that the s-wave superfluid order parameter Δ_s remains unchanged.

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Evaluating the peak position of the p-wave condensate fraction $N_{\mathrm{c}}(S=1, S_z=0)$ from Fig. 3(a), we obtain Fig. 3(b). Recently, the single-component spin-orbit interaction with $0.5 \lesssim \lambda_z/v_{\mathrm{F}} \lesssim 1$ has been realized in a ^{40}K Fermi gas[59]. Keeping this in mind, we find from Fig. 3(b) that the region

$$(k_{\mathrm{F}}a_s)^{-1} \simeq 0, \quad \lambda_z/v_{\mathrm{F}} \simeq 1, \tag{34}$$

is suitable for the preparation of large p-wave pair amplitude. As an example, at $(k_{\mathrm{F}}a_s)^{-1}=0$ and $\lambda/v_{\mathrm{F}}=1$, one obtains

$$\frac{N_{\mathrm{c}}(S=1, S_z=0)}{N} \simeq 0.07. \tag{35}$$

In this case, just after the sudden change of the pairing interaction from the s-wave one to the p-wave one in Eq. (20), we expect the p-wave superfluid state with the p-wave condensate fraction being equal to Eq. (35). This p-wave superfluid state with $\Delta_p(\boldsymbol{p}, S_z=0) \propto p_z$ is just the so-called polar phase discussed in superfluid ^{3}He[41, 42]. As usual[41–43], one can summarize this p-wave superfluid order parameter as

$$\hat{\Delta}(\boldsymbol{p}) = \begin{pmatrix} \Delta_p(\boldsymbol{p}, S_z=1) & \Delta_p(\boldsymbol{p}, S_z=0) \\ \Delta_p(\boldsymbol{p}, S_z=0) & \Delta_p(\boldsymbol{p}, S_z=-1) \end{pmatrix} \sim \begin{pmatrix} 0 & p_z \\ p_z & 0 \end{pmatrix}, \tag{36}$$

where we have ignored the unimportant factor that is nothing to do with the pairing symmetry in the last expression. Since the polar state has not been realized in liquid ^{3}He, a single-component spin-orbit coupled s-wave superfluid Fermi gas would be useful for the realization of this unconventional pairing state.

To actually prepare the p-wave pair amplitude in the parameter region in Eq. (34), the s-wave superfluid phase in this regime must be experimentally accessible. In this regard, we note that T_{c} is also λ_z-independent in the present case. Indeed, when $\lambda_\perp = 0$ and $\lambda_z \neq 0$, we can completely eliminate the λ_z-dependence from the T_{c}-equation (26) as

$$1 = -\frac{4\pi a_s}{m} \sum_{\boldsymbol{p}} \left[\frac{1}{2\tilde{\xi}_{\boldsymbol{p}}} \tanh \frac{\tilde{\xi}_{\boldsymbol{p}}}{2T} - \frac{1}{2\varepsilon_{\boldsymbol{p}}} \right], \tag{37}$$

where $\tilde{\xi}_{\boldsymbol{p}}$ is given below Eq. (32). In the same manner, the non-interacting part N_{free}, as well as the pair correlation function $\Pi(\boldsymbol{q}, i\nu_n)$, in the number equation (27) can be also written in the λ_z-independent forms as, respectively,

$$N_{\mathrm{free}} = 2 \sum_{\boldsymbol{p}} f(\tilde{\xi}_{\boldsymbol{p}}), \tag{38}$$
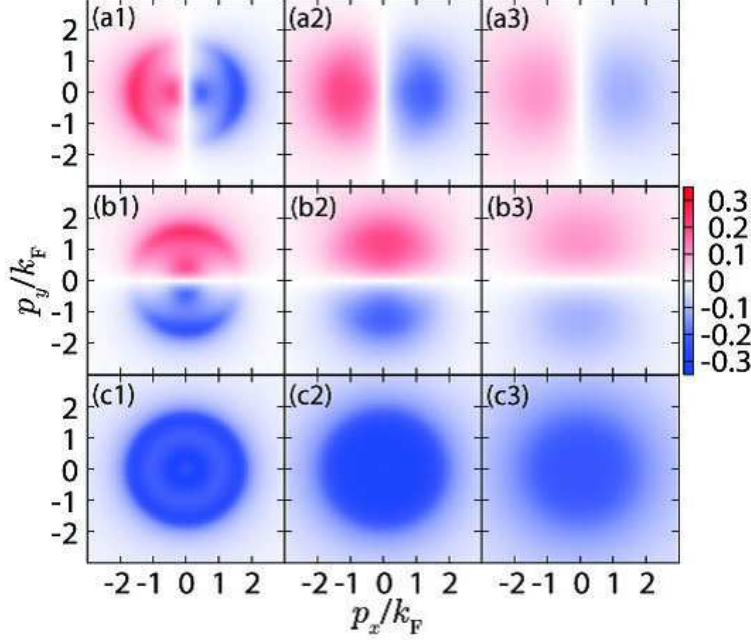
FIG. 4: (Color online) Intensity of the pair amplitude $\Phi(\boldsymbol{p}, S, S_z)$ at $T = 0$ in the case of $\lambda_\perp/v_F = 1$ and $\lambda_z = 0$. The upper and middle panels show $\mathrm{Re}[\Phi(\boldsymbol{p}, S = 1, S_z = 1)]$ and $\mathrm{Im}[\Phi(\boldsymbol{p}, S = 1, S_z = 1)]$, respectively. The lower panels show the s-wave pair amplitude $\Phi(\boldsymbol{p}, S = 0, S_z = 0)$. (a1)-(c1) $(k_F a_s)^{-1} = -1$. (a2)-(c2) $(k_F a_s)^{-1} = 0$. (a3)-(c3) $(k_F a_s)^{-1} = 1$. In these panels, we take $p_z = 0$. In the present case, the p-wave component with $S_z = -1$ is also induced (although we do not show this here), which is simply related to the $S_z = +1$ component as $\Phi(\boldsymbol{p}, S = 1, S_z = -1) = -\Phi^*(\boldsymbol{p}, S = 1, S_z = 1)$.

$$\Pi(\boldsymbol{q}, i\nu_n) = \sum_{\boldsymbol{p}} \frac{1 - f(\tilde{\xi}_{\boldsymbol{p}+\boldsymbol{q}/2}) - f(\tilde{\xi}_{\boldsymbol{p}-\boldsymbol{q}/2})}{\tilde{\xi}_{\boldsymbol{p}+\boldsymbol{q}/2} + \tilde{\xi}_{\boldsymbol{p}-\boldsymbol{q}/2} - i\nu_n}. \tag{39}$$

Thus, the self-consistent solutions of the coupled equations (26) and (27) in Fig. 3(c) is valid for arbitrary values of the spin-orbit coupling strength λ_z. Since current experiments can reach the temperature region far below T_c of a unitary Fermi gas in the absence of a spin-orbit interaction[1–4], the region in Eq. (34) is also accessible within the current experimental technique. In addition, since the superfluid order rapidly grows in the superfluid phase below T_c, when the temperature can be lowered to some extent below T_c, we would be able to obtain the p-wave condensate fraction, the value of which is close to that at $T = 0$ obtained in this paper.

B. two-component spin-orbit interaction ($\lambda_\perp \neq 0$, $\lambda_z = 0$)

When $\lambda_\perp \neq 0$ and $\lambda_z = 0$, the spin-orbit interaction in Eq. (7) consists of the σ_x and σ_y components. In this two-component case, Eq. (18) indicates that the p-wave pair amplitudes with $S_z = \pm 1$ are induced. We show the detailed momentum dependence of the $S_z = 1$ component in the upper and middle panels in Fig. 4. Although we do not show the $S_z = -1$ component, it is simply related to the $S_z = 1$ component as $\Phi(\boldsymbol{p}, S = 1, S_z = -1) = -\Phi^*(\boldsymbol{p}, S = 1, S_z = 1)$. Thus, just after the s-wave interaction is suddenly replaced by the p-wave one in Eq. (20), one has the p-wave superfluid order parameters $\Delta_p(\boldsymbol{p}, S_z = \pm 1)$. Because $|\Delta_p(\boldsymbol{p}, S_z = 1)| = |\Delta_p(\boldsymbol{p}, S_z = -1)|$, this p-wave superfluid phase is just the planar state[41–43]. Under the matrix representation in Eq. (36), one has

$$
\hat{\Delta}(\boldsymbol{p}) \sim \begin{pmatrix} -p_x + ip_y & 0 \\ 0 & p_x + ip_y \end{pmatrix}.
\tag{40}
$$

We briefly note that the planar state has not been realized in liquid ^{3}He.

As in the single-component case, the pair amplitude dominantly appears around the Fermi surface in the weak-coupling BCS regime. When $U_s = 0$, the present two-component spin-orbit interaction gives

$$
\xi_{\boldsymbol{p}}^{\pm} = \frac{(p_\perp \pm m\lambda_\perp)^2}{2m} + \frac{p_z^2}{2m} - \tilde{\mu}_\perp,
\tag{41}
$$

where $\tilde{\mu}_\perp = \mu + m\lambda_\perp^2/2$. At $p_z = 0$, one obtains two Fermi surfaces, both of that are centered at $\boldsymbol{p} = 0$, with the radii,

$$
k_{\mathrm{F}}^{\pm} = \begin{cases} \sqrt{2m\tilde{\mu}_\perp} \pm m\lambda_\perp, & \tilde{\mu}_\perp \geq m\lambda_\perp^2/2, \\ \pm\sqrt{2m\tilde{\mu}_\perp} + m\lambda_\perp, & \tilde{\mu}_\perp < m\lambda_\perp^2/2. \end{cases}
\tag{42}
$$

Figures 4(a1)-(c1) show that both the s-wave and p-wave pair amplitudes are large around these Fermi surfaces, as expected.

In the two-component case, one cannot eliminate the $\lambda_\perp$-dependence from the BCS-Leggett coupled equations (15) and (16). As a result, the s-wave superfluid order parameter Δ_s, as well as the effective Fermi chemical potential $\tilde{\mu}_\perp$, depend on the spin-orbit coupling strength $\lambda_\perp$, as shown in Figs. 5(a) and (c). Because of this, we see in Fig. 5(b) and (d) that, not only the p-wave condensate fraction $N_c(S = 1, S_z = \pm 1)$, but also the total condensate fraction N_c^{t} is affected by the spin-orbit interaction, which is different from the
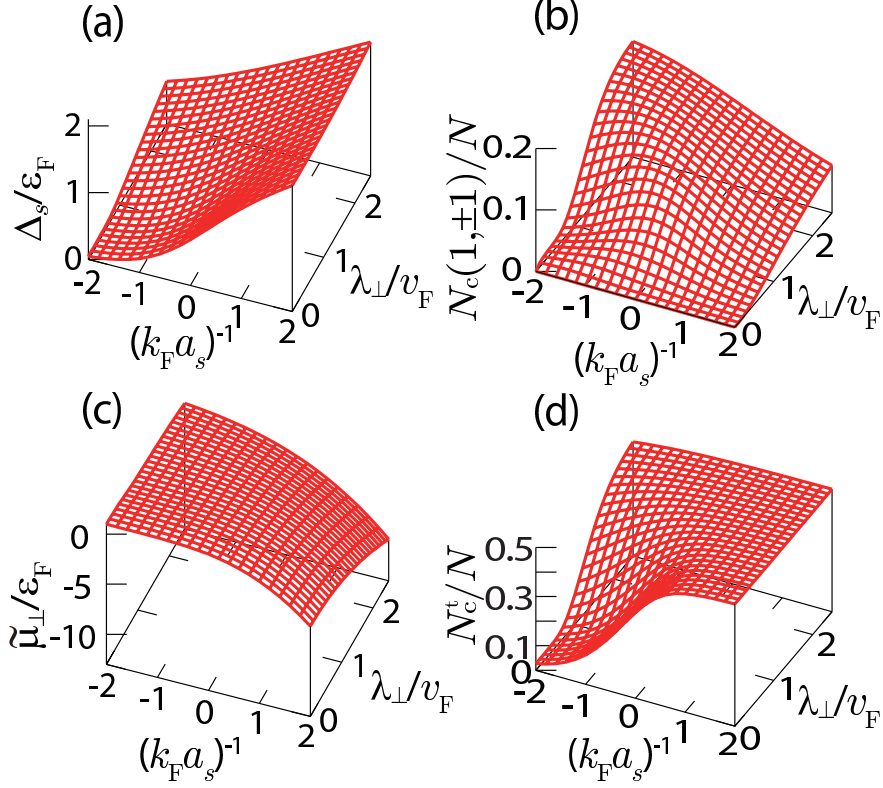
FIG. 5: (Color online) (a) Calculated s-wave superfluid order parameter Δ_s and (c) effective chemical potential $\tilde{\mu}_\perp = \mu + m\lambda_\perp^2/2$ at $T = 0$ in the case of two-component spin-orbit interaction $(\lambda_\perp \neq 0, \lambda_z = 0)$. (b) p-wave condensate fraction $N_c(S = 1, S_z = \pm 1)$. (d) Total condensate fraction N_c^t.

single-component case discussed in the previous subsection. The $\lambda_\perp$ dependence of N_c^t can also be confirmed by analytically carrying out the momentum summation in Eq. (25) as

$$\frac{N_c^t}{N} = \frac{3\pi\Delta_s}{16\sqrt{2}\varepsilon_F^{3/2}} \left[\sqrt{\mu + \sqrt{\mu^2 + \Delta_s^2}} + \frac{\sqrt{m}\lambda_\perp}{2} \arccos \frac{\sqrt{\mu^2 + \Delta_s^2} - m\lambda_\perp^2/2}{\sqrt{\tilde{\mu}_\perp^2 + \Delta_s^2}} \right]. \quad (43)$$

Figure 5(d) shows that the total condensate fraction N_c^t in the BCS side $((k_F a_s)^{-1} \lesssim 0)$ is remarkably enhanced by the spin-orbit interaction, to be comparable to the value in the BCS regime with $\lambda_\perp = 0$. Reflecting this enhancement, we find from Fig. 5(b) that the p-wave component $N_c(S = 1, S_z = \pm 1)$ is also enhanced in the BCS regime, when $\lambda_\perp$ is large. Thus, as shown in Fig. 6(a), the peak position of $N_c(S = 1, S_z = \pm 1)$ shifts to the BCS side, compared to the single component case shown in Fig. 3(b).

Although the superfluid phase transition temperature T_c is usually low in the weak-
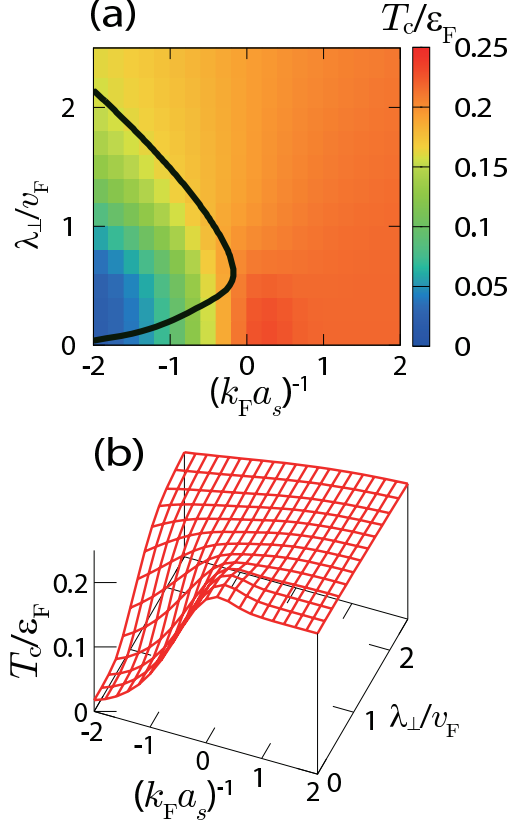
FIG. 6: (Color online) (a) Peak position of p-wave condensate fraction $N_\mathrm{c}(S = 1, S_z = \pm 1)$ at $T = 0$ in the case of two-component spin-orbit interaction. The background intensity represents the magnitude of T_c. For convenience, we also show the three-dimensional plot of T_c in panel (b).

coupling BCS regime, Fig. 6(b) shows that T_c around the "optimal region" (peak line in Fig. 6(a)) in the BCS regime is also enhanced by the spin-orbit interaction. Since we need to reach the s-wave superfluid phase in order to prepare the p-wave pair amplitude, this enhancement of T_c is favorable for our purpose. We briefly note that the enhancement of T_c by a Rashba-type spin-orbit interaction has recently been pointed out[71].

The reason for the enhancement of the condensate fraction, as well as T_c, in the weak-coupling BCS regime is the formation of two-body bound molecules[65, 71, 73–76], which are also referred to as rashbons in the literature. In the present case, the degeneracy of the lowest energy level in the lower band $\xi_{\boldsymbol{p}}^-$ in Eq. (41) leads to a two-dimensional-like density of states around the bottom of this band. This "low-dimensional effect" stabilizes a two-body bound state (rashbon), even in the weak-coupling BCS regime[65, 71, 73–76], where such a two-body bound state does not appear in the ordinary three-dimensional system.
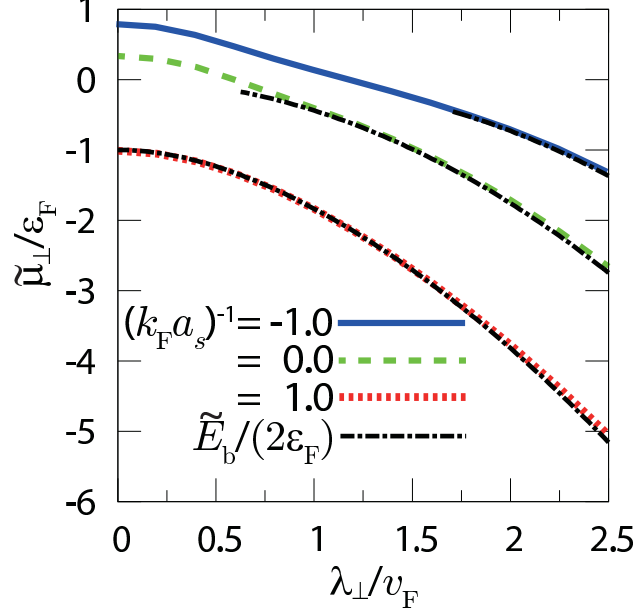
FIG. 7: (Color online) Calculated effective chemical potential $\tilde{\mu}_\perp = \mu + m\lambda_\perp^2/2$ at T_c as a function of the spin-orbit coupling strength $\lambda_\perp$. $\tilde{E}_b/2 = E_b/2 + m\lambda_\perp^2/2$, where E_b is the binding energy of a two-body bound state, determined from Eq. (44).

Then, the superfluid phase transition in the strong spin-orbit coupling regime is dominated by the BEC of rashbons, giving a high T_c even in the BCS regime. This is similar to the case of the ordinary strong-coupling BEC regime of an ultracold Fermi gas[29–39], where the superfluid phase transition is dominated by tightly bound molecules that are formed by a strong pairing interaction. Indeed, as shown in Fig. 7, when the spin-orbit coupling strength $\lambda_\perp$ increases, the effective Fermi chemical potential $\tilde{\mu}_\perp = \mu + m\lambda_\perp^2/2$ at T_c becomes negative, to approach $\tilde{E}_b/2 = E_b/2 + m\lambda_\perp^2/2$, where E_b is the binding energy of a two-body bound state, determined from the equation[65, 71, 73–76],

$$1 = -\frac{4\pi a_s}{m} \sum_{\boldsymbol{p}} \left[\frac{1}{2} \sum_{\alpha=\pm} \frac{1}{2(\varepsilon_{\boldsymbol{p}} + \alpha\lambda_\perp p_\perp) - E_b} - \frac{1}{2\varepsilon_{\boldsymbol{p}}} \right] \quad (E_b < -m\lambda_\perp^2/2). \qquad (44)$$

Since the chemical potential physically describes energy to add a particle to the system, Fig. 7 indicates that most Fermi atoms form two-body bound molecules in the strong spin-orbit coupling regime.

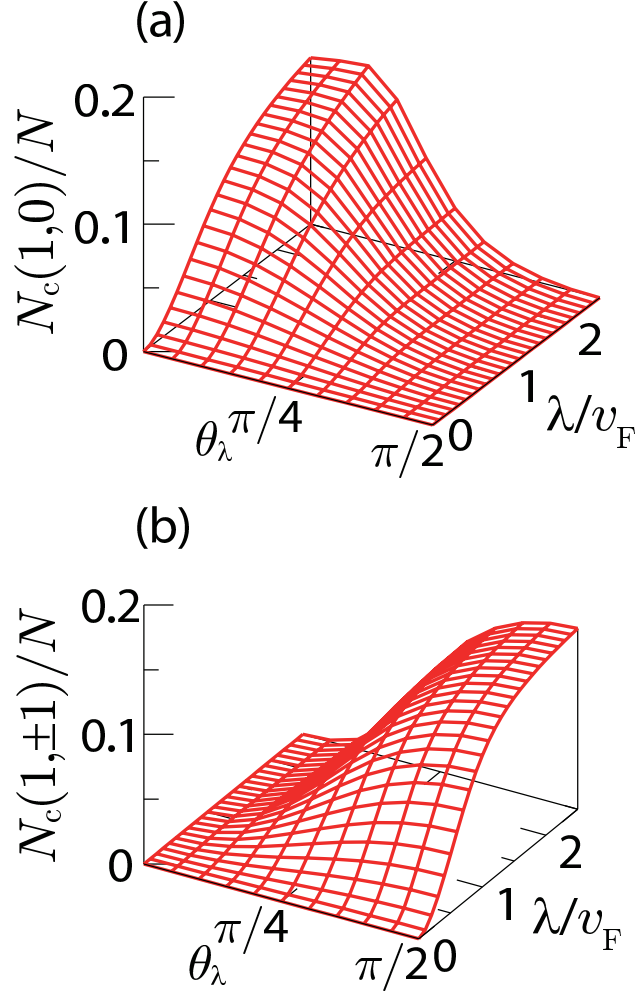$$(k_{\mathrm{F}}a_s)^{-1} = 0$$

(a)

(b)

FIG. 8: (Color online) Calculated p-wave condensate fraction at $T = 0$ in the case of three component spin-orbit interaction $(\lambda_\perp, \lambda_z) = \lambda(\sin\theta_\lambda, \cos\theta_\lambda)$. We set $(k_{\mathrm{F}}a_s)^{-1} = 0$. (a) $N_{\mathrm{c}}(S = 1, S_z = 0)$. (b) $N_{\mathrm{c}}(S = 1, S_z = \pm 1)$. In this figure, $\theta_\lambda = 0$ and $\theta_\lambda = \pi/2$, respectively, correspond to the single-component and two-component spin-orbit interaction, discussed in the previous subsections.

C. Three component spin-orbit interaction ($\lambda_\perp \neq 0,\ \lambda_z \neq 0$)

When the spin-orbit interaction has all the σ_x, σ_y, and σ_z components $[(\lambda_\perp, \lambda_z) = \lambda(\cos\theta_\lambda, \sin\theta_\lambda)\ (0 \leq \theta_\lambda \leq \pi/2)]$, all the p-wave condensate fractions, $N_{\mathrm{c}}(S = 1, S_z = \pm 1, 0)$, are induced, as shown in Fig. 8.

When the spin-orbit interaction is isotropic ($\lambda_\perp = \lambda_z$ or $\theta_\lambda = \pi/4$), Figs. 9(a) and (b)
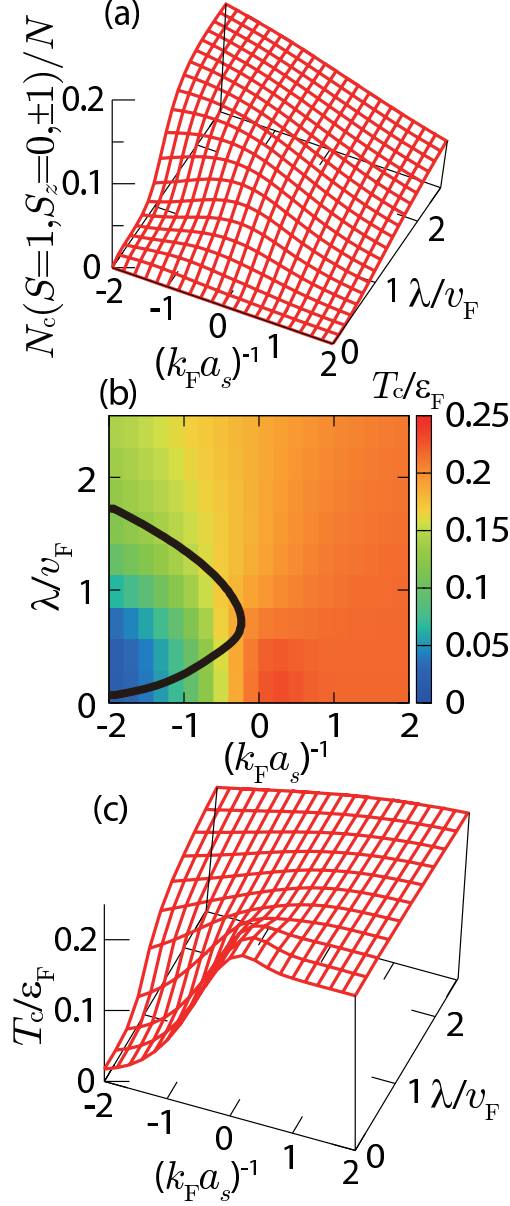
FIG. 9: (Color online) (a) p-wave condensate fraction $N_c(S = 1, S_z = 0, \pm 1)$ at $T = 0$ in the case of isotropic spin-orbit interaction $\lambda_\perp = \lambda_z = \lambda/\sqrt{2}$ ($\theta_\lambda = \pi/4$). (b) Peak position of $N_c(S = 1, S_z = 0, \pm 1)$ evaluated from panel (a). The background intensity shows T_c, which is also shown in panel (c) for clarity.

indicate that the BCS side is suitable for our purpose. Although we do not explicitly show the result here, the total condensate fraction, which is analytically given by

$$\frac{N_c^t}{N} = \frac{3\pi\Delta_s}{16\sqrt{2}\varepsilon_F^{3/2}}\sqrt{\tilde{\mu} + \sqrt{\tilde{\mu}^2 + \Delta_s^2}}\left[1 + \frac{m\lambda^2/4}{\sqrt{\tilde{\mu}^2 + \Delta_s^2}}\right], \tag{45}$$

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(where $\tilde{\mu} = \mu + m\lambda^2/4$) is also enhanced in the BCS side by the rashbon formation discussed in the previous subsection. In addition, $T_{\rm c}$ is also enhanced by the same mechanism (See Fig. 9(c)), so that this regime is still considered to experimentally be accessible. In this case, when the s-wave pairing interaction is suddenly replaced by the p-wave one in Eq. (20), as the initial p-wave state, we can prepare the BW (Balian-Werthamer) superfluid order parameter[41–43], having the form

$$\hat{\Delta}(\boldsymbol{p}) \sim \begin{pmatrix} -p_x + ip_y & p_z \\ p_z & p_x + ip_y \end{pmatrix}. \tag{46}$$

The BW state has been realized in superfluid ^{3}He[41].

IV. SUMMARY

To summarize, we have discussed a possible idea to realize a p-wave superfluid Fermi gas. In contrast to the ordinary approach where one tries to cool down a Fermi gas with a strong p-wave pairing interaction, our idea consists of two stages. In the first stage, we only prepare a p-wave pair amplitude by, not using a p-wave interaction, but using the phenomenon that a p-wave pair amplitude is induced in a s-wave Fermi superfluid with broken inversion symmetry by an antisymmetric spin-orbit interaction. Then, in the second stage, we suddenly replace the s-wave interaction by a p-wave one, to produce the p-wave superfluid order parameter that is essentially given by the product of the p-wave interaction and the p-wave pair amplitude which has been prepared in the first stage. In this paper, we have assessed the first stage of our idea, by evaluating how large the p-wave pair amplitude can be induced in a spin-orbit coupled s-wave superfluid state. We clarified the region where a large p-wave pair amplitude is obtained, in the phase diagram of a Fermi gas with respect to the strengths of the s-wave pairing interaction and the spin-orbit coupling. Within the framework of the NSR theory, we also confirmed that the s-wave superfluid phase in this optimal region is accessible within the current experimental technique.

The key of our idea is that the pairing symmetry of a Fermi superfluid is just the symmetry of the superfluid order parameter, which is essentially given by the product of a pairing interaction and a pair amplitude. Thus, even when a p-wave pair amplitude is induced in a s-wave superfluid Fermi gas, the system is still in the s-wave superfluid state because of the vanishing p-wave interaction. This also implies the possibility that one may artificially

produce a p-wave Fermi superfluid by separately preparing a p-wave pair amplitude and a p-wave interaction. Our idea just uses this possibility, that is, we separately prepare these two quantity by using two sophisticated techniques developed in cold atom physics: the artificial gauge field and the tunable pairing interaction associated with a Feshbach resonance.

At present, all the experiments aiming a p-wave superfluid Fermi gas uses a p-wave pairing interaction from the beginning. While this approach is straightforward, it seems suffering from the short lifetime of p-wave molecules, as well as the particle loss by the p-wave interaction. In contrast, since our idea does not rely on the p-wave interaction in the first stage, this difficulty can be avoided to some extent. In addition, since we can start from the situation with a finite value of the p-wave superfluid order parameter and a finite value of the p-wave condensate fraction, even when the p-wave interaction eventually destroys this superfluid state, we expect to be able to realize a p-wave superfluid Fermi gas for a while, after the p-wave interaction is introduced.

In this paper, we have only discussed the first stage of our idea. Since the p-wave superfluid state which is artificially produced would be in the non-equilibrium state, as the next step, we need to examine how this p-wave state relaxes into the thermodynamically stable state, after the p-wave interaction is turned on. In addition, since there are various p-wave superfluid phases, such as the BW phase and polar phase, the initial p-wave superfluid state may not be the most stable state for a given strength of a p-wave interaction and a given value of temperature. Thus, in order to soon reach the thermodynamically stable state, one need to clarify the optimal condition for the initial p-wave pairing state. In our future papers, we will discuss these interesting problems existing in the second stage of our idea. Since both the spin and orbital degrees of freedom are active in a p-wave Fermi superfluid, the realization of a p-wave superfluid Fermi gas would enable us to discuss much richer physics than the case of the s-wave superfluid Fermi gas. Since current experiments toward the realization of a unconventional superfluid Fermi gas faces various difficulties, our results would be useful for the exploration of a route to accomplish this exciting challenge in cold Fermi gas physics.

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